Therefore, this point, which is definitely associated with the moving system, has the property of constant acceleration parallel to the inclined plane. Equation (12) can be integrated at once giving $d\xi/dt$ and ξ as explicit functions of the time, quite independently of θ. On the other hand, $\eta = (msR + m_1y_1)/(ms + m_1)$; hence,

$$\frac{d^2\eta}{dt^2} = \frac{m_1}{ms + m_1} \cdot \frac{d^2y_1}{dt^2},$$

which is, in general, a function of θ.

An important special case of the motion of the system of bodies is brought out by (6). This equation is satisfied for all time if θ maintains the constant value θ' given by

$$\tan \theta' = \frac{mk^2}{mk^2 + (m + m_1)R^2} \cdot \tan \alpha; \tag{13}$$

for then $d^2\theta/dt^2 = d\theta/dt = 0$. Relation (13) shows that $\alpha > \theta' > 0$, as in the diagram. Under these conditions (4) and (5) give, respectively, $dx_1/dt = dx/dt$ and $dy_1/dt = 0$. Consequently, if we start with the centers of the inner and outer bodies at the respective points $(x_0 + \rho \sin \theta', R - \rho \cos \theta')$ and (x_0, R), and impart equal linear velocities parallel to the incline, the inner body will not oscillate relative to its constraining wall but will maintain the constant angular position θ' while the shell slips under it. The equations of motion of the bodies are now easily shown to be

$$(ms + m_1)\frac{d^2x}{dt^2} = (ms + m_1)\frac{d^2x_1}{dt^2} = g(m + m_1) \sin \alpha,$$

$$(ms + m_1)\frac{dx}{dt} = (ms + m_1)\frac{dx_1}{dt} = g(m + m_1) \sin \alpha \cdot t + (ms + m_1)v_0,$$

$$(ms + m_1)x = \tfrac{1}{2}g(m + m_1) \sin \alpha \cdot t^2 + (ms + m_1)(v_0t + x_0),$$

$$(ms + m_1)x_1 = \tfrac{1}{2}g(m + m_1) \sin \alpha \cdot t^2 + (ms + m_1)(v_0t + x_0 + \rho \sin \theta').$$

Finally, if we introduce a new variable angle λ, defined by the equation $\lambda = \theta - \theta'$, and make use of (13) we can change the expression under the radical in the denominator of the integral in (8) to the form

$$\rho(ms + m_1 \sin^2 \theta_0)\omega_0^2 + 4g(ms + m_1) \cos \alpha \sec \theta' \sin \tfrac{1}{2}[(\theta' - \theta_0) + \lambda] \sin \tfrac{1}{2}[(\theta' - \theta_0) - \lambda],$$

which assumes the same value when λ is assigned values which are numerically equal but of opposite sign. Unfortunately for the analysis, the numerator of the integrand does not possess this kind of symmetry.

No other correct solution of this problem was received. Should any one integrate equation (8) we shall be glad to publish the result.—EDITORS.

NUMBER THEORY.

223. (October, 1914) Proposed by T. E. MASON, Purdue University.

Show that

$$\frac{(rst)!}{t!(s!)^t(r!)^{st}}$$

is an integer, r, s, and t being positive integers. Generalize to the case of n integers, $r, s, t, u, \cdots$.

[Carmichael's *Theory of Numbers*, page 28.]

SOLUTION BY FRANK IRWIN, University of California.

Suppose we have rst objects, and let us divide them into t classes of rs objects each, then each class into s sub-classes of r objects each, and let us call each such classification, without any reference to order, a "classification" *par excellence*. We assert that the total number of such classifications is

$$\frac{(rst)!}{t!(s!)^t(r!)^{st}},$$

which expression is, consequently, an integer.

For, let us set up an order among the classes, among the sub-classes in each class, and among the objects in each sub-class. Then from any given classification we may, by permutations of the objects which make changes in the order merely, but do not change the constituents of the classes and sub-classes—which, then, leave the classification as such the same—we may, by such operations, obtain various permutations of the totality of the rst objects. First, without changing the order of the classes or sub-classes, we may permute among themselves the objects in each sub-class. For any one sub-class this may be done in $r!$ ways, and, therefore, for all the st sub-classes in $(r!)^{st}$ ways. Next, we may permute among themselves the sub-classes in each class. For any one class this may be done in $s!$ ways; for all the t classes, then, in $(s!)^t$ ways. Finally, we may permute the classes in $t!$ ways.

Combining these various kinds of permutations, we see that each classification gives rise to $t!(s!)^t(r!)^{st}$ permutations of the rst objects. Since each of these latter permutations may be derived, by the process explained, from some classification or other, and two different classifications cannot give rise to the same permutation, it follows that the number of permutations of the rst objects is $t!(s!)^t(r!)^{st}$ times the number of classifications; that is, since there are $(rst)!$ permutations there are, as was to be shown, $(rst)!/t!(s!)^t(r!)^{st}$ classifications.

The argument may evidently be generalized to show (with a change in the notation) that

$$\frac{(r_1r_2\cdots r_n)!}{r_1!(r_2!)^{r_1}(r_3!)^{r_1r_2}\cdots(r_n!)^{r_1r_2\cdots r_{n-1}}}$$

is an integer.

228. Proposed by HERMON C. KATANIK,* Indianapolis, Ind.

Deduce a formula for the difference between any two squares, and thus show that (1) The difference between any two consecutive squares is of the form $2n+1$; (2) The difference between any two squares is even or odd according to whether they are separated by an odd or even number of squares; (3) The differences of the squares of the consecutive terms of any arithmetic progression form another arithmetic progression.

SOLUTION BY WALTER C. EELLS, Whitman College.

Let T_i be the ith term. Then $T_n = n^2$, $T_{n+k} = (n+k)^2$.

$$T_{n+k} - T_n = (n+k)^2 - n^2 = (n+k-n)(n+k+n) = k(2n+k).$$

(1) If $k=1$, $T_{n+1} - T_n = (2n+1)$.

(2) When separated by an odd number of squares $T_{n+2k'} - T_n = 2k'(2n+2k')$, which is even. When separated by an even number of squares $T_{n+2k'+1} - T_n = (2k'+1)(2n+2k'+1)$, which is odd.

(3) Since k is constant for any given arithmetic series, $T_{n+mk} - T_{n+(m-1)k}$ is constant and is the constant difference for another arithmetic series.

Also solved by HORACE OLSON and HERBERT N. CARLETON.

QUESTIONS AND DISCUSSIONS.

SEND ALL COMMUNICATIONS TO U. G. MITCHELL, University of Kansas, Lawrence, Kans.

A number of questions published in this department have been standing for some time without having been answered. We are re-publishing these in full this month in the hope that some of our readers may thereby be stimulated to send in suitable replies.

12. In view of the notation used by Professor Slobin in his "Note on Certain Algebraic Equations" published in the MONTHLY for April, 1914, pages 113–115, a discussion would be desirable as to the best notation for complex roots in general, and in particular for eliminating the conspicuous ambiguities introduced by the notation above cited.

15. We are in receipt of the following communication from Mr. W. E. Heal, of Washington, D. C.: "In the Proceedings of the Royal Society of Edinburgh, Vol. VII, p. 144, in some mathematical notes by Professor P. G. Tait, it is stated:

* Deceased since proposing this problem.